

9/6/17 "Nobody can go back and start a new beginning, but anyone can start today and make a new ending." -Maria Robinson

HW: Finish "Understanding Limits Graphically and Numerically" packet

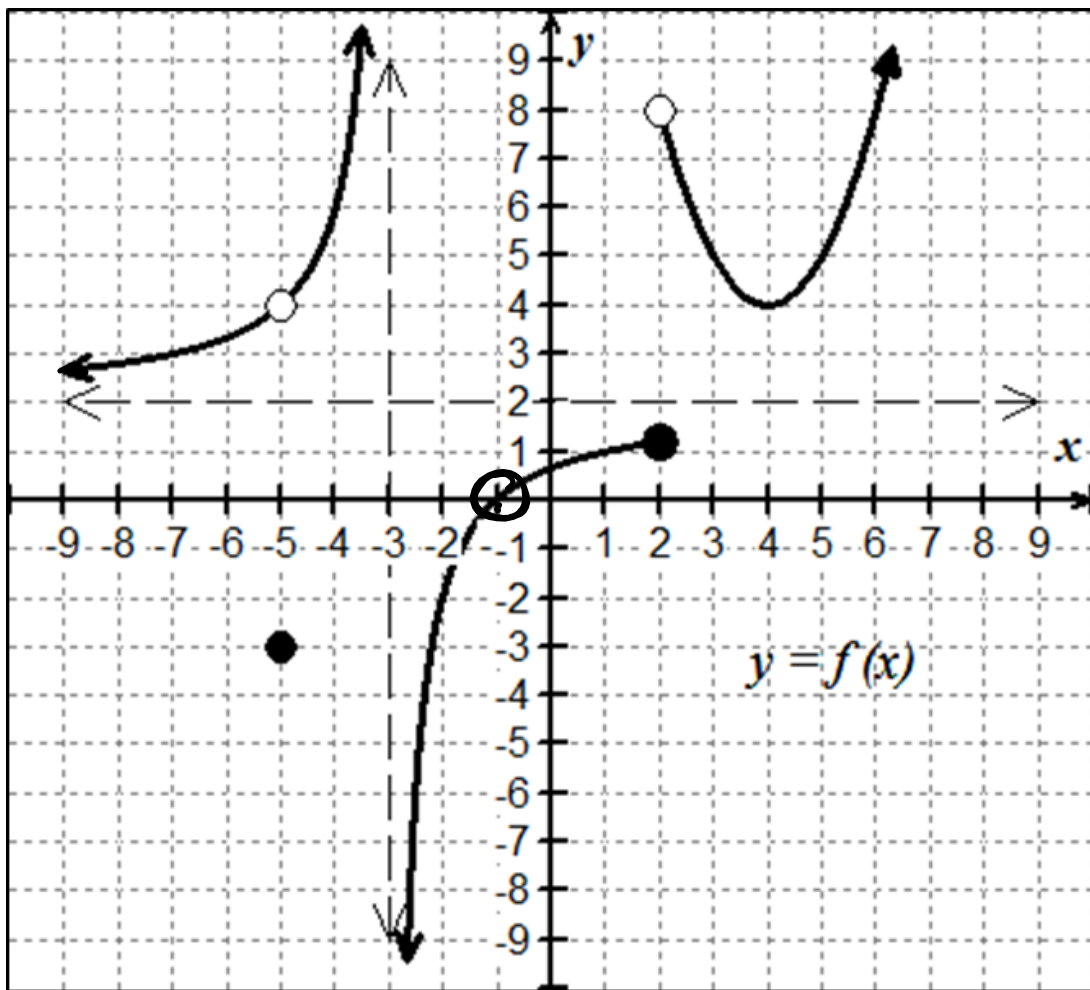
AIM: Understanding Limits Graphically

Warm Up:

Think about "What is a limit?"

Understanding the Limit Graphically and Numerically

Consider the graph of the function $f(x)$, graphed below:

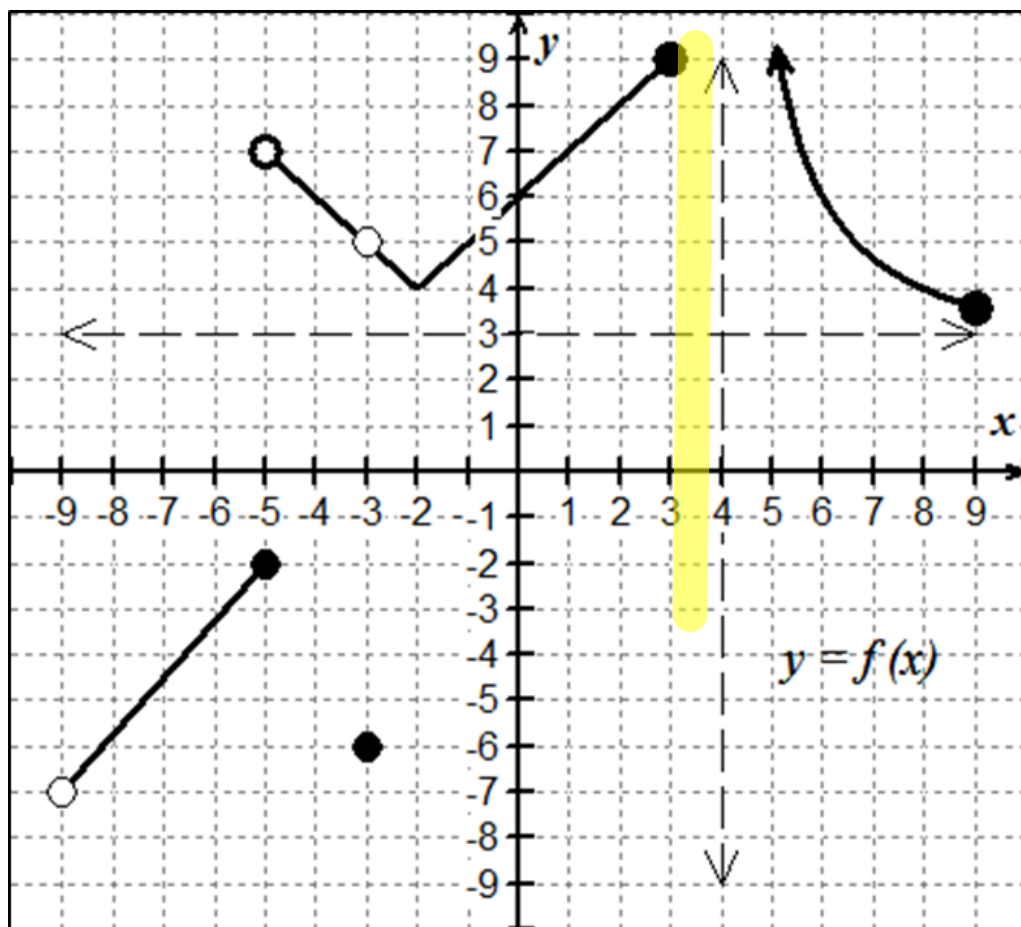


Using the graph, find the value of each of the following limits.
If a limit does not exist, explain why.

A.) $\lim_{x \rightarrow -3^-} f(x)$ ∞	B.) $\lim_{x \rightarrow -5} f(x)$ 4	C.) $\lim_{x \rightarrow 4} f(x)$ 4
D.) $\lim_{x \rightarrow 2^+} f(x)$ 8	E.) $\lim_{x \rightarrow 2^-} f(x)$ 1	F.) $\lim_{x \rightarrow 2} f(x)$ DNE
G.) $\lim_{x \rightarrow -1} f(x)$ 0	H.) $\lim_{x \rightarrow -\infty} f(x)$ 2	I.) $\lim_{x \rightarrow \infty} f(x)$ ∞



Now you give it a try. Consider the graph shown below to find the value of each of the following limits. If a limit does not exist, explain why.



A. $\lim_{x \rightarrow -5^+} f(x)$ 7	B. $\lim_{x \rightarrow -2} f(x)$ 4	C. $\lim_{x \rightarrow -3} f(x)$ 5
D.) $\lim_{x \rightarrow 3^+} f(x)$ DNE (no function)	E.) $\lim_{x \rightarrow 3^-} f(x)$ 9	F.) $\lim_{x \rightarrow -5^-} f(x)$ -2
G.) $\lim_{x \rightarrow 0} f(x)$ 6	H.) $\lim_{x \rightarrow -9} f(x)$ DNE (no left side)	I.) $\lim_{x \rightarrow 4^+} f(x)$ ∞



Limits are the “backbone” of understanding that connects algebra and geometry to the mathematics of calculus. In basic terms, a limit is just a statement that tells you what height a function *INTENDS TO REACH* as you get close to a specific x -value. Recall from Pre-Calculus that you evaluated three types of limits. Complete the table below:

PROPER LIMIT NOTATIONS		
TYPE OF LIMIT	PROPER NOTATION	VERBALLY:
Right-hand limit	$\lim_{x \rightarrow c^+} f(x)$	limit of $f(x)$ as x goes to c from right
Left-hand limit	$\lim_{x \rightarrow c^-} f(x)$	limit as x goes to c from left
General limit	$\lim_{x \rightarrow c} f(x)$	limit as x goes to c

Consider the function shown below.

Say you want to find $\lim_{x \rightarrow 4^+} f(x)$, the positive sign in the limit notation indicates a right-hand limit. If you think of the function as a highway and imagine you are traveling along the graph of $f(x)$ toward $x = 4$ FROM THE RIGHT, NOT TO THE RIGHT, and you stop at the vertical line $x = 4$, the y -value where you stop is 3. Therefore, $\lim_{x \rightarrow 4^+} f(x) = 3$.

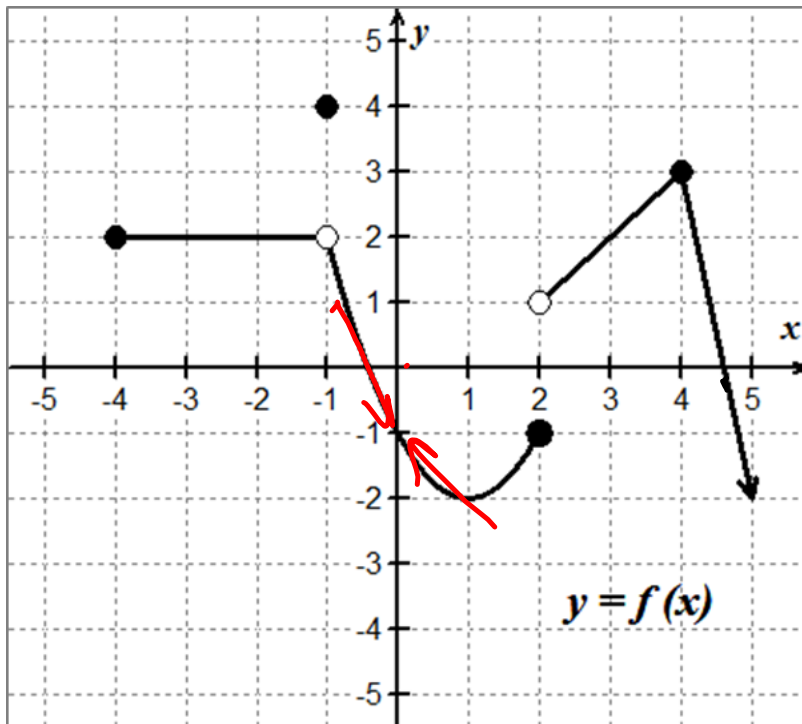


Figure 1-1

You will use this graph to explore the limits for the problems on the next page.





EX #1: Use *Figure 1-1* to find the function values and evaluate each of the following limits:

1. $f(2)$ -1	2. $f(-1)$ $= 4$
3. $\lim_{x \rightarrow 4^-} f(x)$ 3	4. $\lim_{x \rightarrow 2^+} f(x)$ 1
5. $\lim_{x \rightarrow 2^-} f(x)$ -1	6. $\lim_{x \rightarrow -1^+} f(x)$ 2
7. $\lim_{x \rightarrow -1^-} f(x)$ 2	8. $\lim_{x \rightarrow -4^+} f(x)$ 2
9. $\lim_{x \rightarrow -4^-} f(x)$ DNE	10. $\lim_{x \rightarrow -1} f(x)$ 2
11. $\lim_{x \rightarrow 2} f(x)$ DNE	12. $\lim_{x \rightarrow 5} f(x)$ -2
13. $\lim_{x \rightarrow 0} f(x)$ -1	14. $\lim_{x \rightarrow 1} f(x)$ -2

EX #2: THINK ABOUT THIS!

If we think of the function as a highway, then the point at $(2, -1)$ could be considered the end of the road, while the point at $(-1, 2)$ is more like a “pothole.” How would you describe the points located at

$(2, 1)$ Dead End

$(4, 3)$ Turn in road

Hopefully, this analogy gives you a visual reference for understanding limits from a graphical approach. Let's get a little more formal with our definition now.

When finding limits, ask yourself, “What is happening to y as x gets close to a certain number?” You are finding the **y -value** for which the function is approaching as x approaches c .

LIMIT EXISTENCE THEOREM:

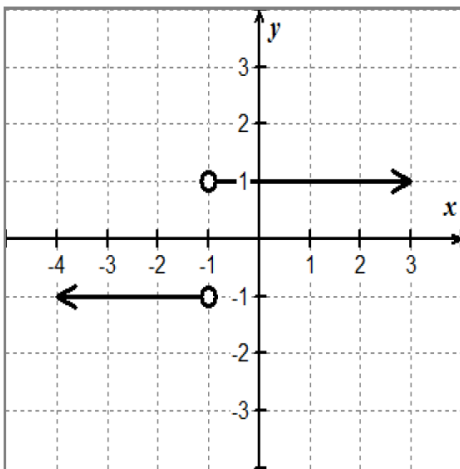
$\lim_{x \rightarrow c} f(x)$ exists if and only if

$$\lim_{x \rightarrow c^-} f(x) = \lim_{x \rightarrow c^+} f(x) = L \quad \left(\begin{array}{l} L \text{ is} \\ \text{a real} \\ \# \end{array} \right)$$

Verbally: The limit as x approaches c on $f(x)$ will exist if and only if the limit as x approaches c from the left is equal to the limit as x approaches c from the right.

EX #3: LIMITS CAN FAIL TO EXIST IN THREE SITUATIONS:

CASE 1: limits differ from left and right



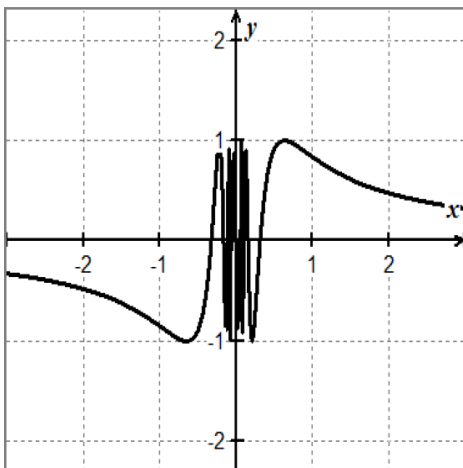
Justify why the limit does not exist at $x = -1$ for $f(x) = \frac{|x+1|}{x+1}$

$$\lim_{x \rightarrow -1^-} f(x) = -1$$

$$\lim_{x \rightarrow -1^+} f(x) = 1$$

Not
Same

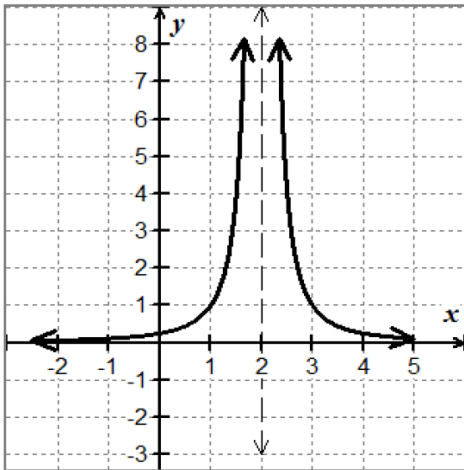
CASE 2: Oscillating behavior



Justify why the limit does not exist at $x = 0$ for $f(x) = \sin\left(\frac{1}{x}\right)$

$$\lim_{x \rightarrow 0^+} f(x) < 0$$

$$\lim_{x \rightarrow 0^-} f(x) > 0$$

CASE 3: limits of ∞ or $-\infty$ 

Justify why the limit does not exist at $x = 2$ for $f(x) = \frac{1}{(x-2)^2}$

$$\lim_{x \rightarrow 2^-} f(x) = \infty$$

$$\lim_{x \rightarrow 2^+} f(x) = \infty$$

Not
Real

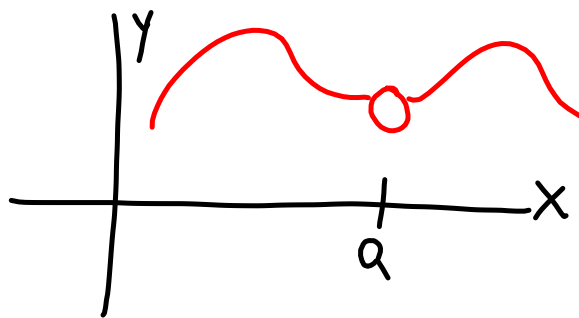
EX #4: YOU OWN IT! In the box below, complete the sentence in your own words.

In order for the GENERAL LIMIT to exist, the function:

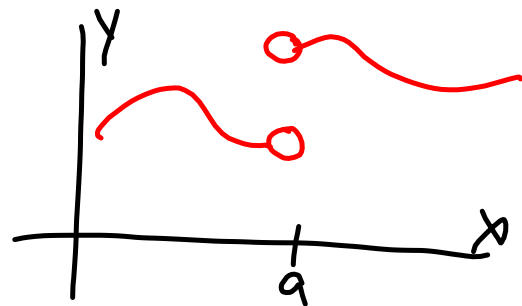
From Left = From right

EX #5: Sketch a graph to satisfy each set of conditions.

1. $f(a)$ is undefined (open circle)
2. $x = a$ is a point discontinuity
3. $\lim_{x \rightarrow a} f(x)$ exists



- ✓ 1. $\lim_{x \rightarrow a} f(x)$ DNE
- ✓ 2. $x = a$ is a jump discontinuity
3. $f(a)$ is undefined



EX #6: Finding limits from a table of values

Now consider the function $f(x) = \frac{x-3}{x^2+2x-15}$.

Complete the table below to find the limit as $x \rightarrow 3$.

x	2.9	2.99	2.999	3	3.001	3.01	3.1
$f(x)$							

Based on your analysis, what are the values of each of the limits below?

$\lim_{x \rightarrow 3^-} f(x) =$	$\lim_{x \rightarrow 3^+} f(x) =$	$\lim_{x \rightarrow 3} f(x) =$
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